Dynamic Macroeconomics: Fiscal and Monetary Policy

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By Hans Brems

Abstract

The paper adds government to a neoclassical growth model. With government come money, government expenditure, taxes, and the government budget constraint. The resulting model is solved for the growth rates of all its variables. The growth rate of the money supply never appears in solutions for real variables but always appears in solutions for nominal ones. The model is also solved for the level of the real rate of interest. Whether financed by a higher tax rate or by a more rapidly growing money supply, larger government purchases will raise the level of the real rate of interest.

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I. INTRODUCTION

1. Purpose and Framework

For all its simplicity, Solow's (1956) neoclassical growth model simulated a century of U.S. economic growth remarkably well. But it was a model of a purely private economy: no government purchase of goods, no taxes, and no money supply. The purpose of the present paper is to add government.

With government comes money, and with money come prices and a rate of interest. Indeed under inflation come two rates of interest, a nominal one and a real one, as Turgot [1769-1770 (1922: 75-76)], Fisher (1896), and Mundell (1971) emphasized. With government come taxes. Solow never optimized his capital stock. We shall optimize

it and find inflation, interest, and taxes important for such optimi-
zation, as Feldstein (1976) had found.

   With government come crowding-out and inflation--but also the
fiscal-policy and monetary-policy instruments needed to control them.
Policy conclusions may be drawn, and we shall drawn them.

   We shall use the following notation.

2. Variables

C ≡ physical consumption

D ≡ desired holding of money

G ≡ physical government purchase of goods

g ≡ proportionate rate of growth

I ≡ physical investment

k ≡ present gross worth of another physical unit of capital stock

κ ≡ physical marginal productivity of capital stock

L ≡ labor employed

n ≡ present net worth of another physical unit of capital stock

P ≡ price of good

R ≡ tax revenue

r ≡ before-tax nominal rate of interest

ρ ≡ aftertax real rate of interest

S ≡ physical capital stock

w ≡ money wage rate

X ≡ physical output

Y ≡ money national income

y ≡ money disposable income

3. Parameters

  a ≡ multiplicative factor of production function

α,β ≡ exponents of a production function

  c ≡ propensity to consume

  F ≡ available labor force

  λ ≡ proportion employed of available labor force

  M ≡ supply of money

  m ≡ multiplicative factor of demand-for-money function

  μ ≡ exponent of demand-for-money function

  T ≡ tax rate

    The model will include derivatives with respect to time; hence it
is dynamic. All parameters are stationary except a, F, and M, whose
growth rates are stationary.

II. THE MODEL

Define the proportionate rate of growth of variable v as

$$g_v \equiv \frac{dv}{dt}\frac{1}{v} \tag{1}$$

Define investment as

$$I \equiv g_S S \tag{2}$$

Let an aggregate production function be of Cobb-Douglas form:

$$X = aL^{\alpha}S^{\beta} \tag{3}$$

where $0 < \alpha < 1$; $0 < \beta < 1$; $\alpha + \beta = 1$; and $a > 0$.

Let purely competitive entrepreneurs maximize their profits with respect to labor employed. Let profits be taxed at the rate T. Before-tax profits equal market value of output minus labor cost minus capital cost, and aftertax profits equal the constant 1 - T

<u>times</u> before-tax profits. Before-tax and aftertax profits are maximized, then, at the same labor employed L, i.e., where the real wage rate equals the physical marginal productivity of labor:

$$\frac{w}{P} = \frac{\partial X}{\partial L} = \alpha \frac{X}{L} \tag{4}$$

Rearrange (4) and write the neoclassical mark-up-pricing equation

$$P = \frac{wL}{\alpha X} \tag{5}$$

saying that neoclassical price P equals per-unit labor cost wL/X marked up in the proportion 1/α.

Define physical marginal productivity of capital stock as

$$\kappa \equiv \frac{\partial X}{\partial S} = \beta \frac{X}{S} \tag{6}$$

Profits were taxed at the rate T. At time t, then, aftertax marginal value productivity of capital stock is $(1 - T)\kappa(t)P(t)$.

Let there be a market in which money may be placed or borrowed at the stationary nominal rate of interest r. Let nominal interest earnings be taxed and nominal interest expense be tax-deductible at the rate T. Then money may be placed or borrowed at the aftertax rate $(1 - T)r$. Let that rate be applied when discounting future cash flows. As seen from the present time τ, then, aftertax marginal value productivity of capital stock is $(1 - T)\kappa(t)P(t)e^{-(1 - T)r(t - \tau)}$. Define present gross worth of another physical unit of capital stock as the present worth of all future aftertax marginal value productivities over its entire useful life. Let capital stock be immortal: $u \rightarrow \infty$. Then

$$k(\tau) \equiv \int_{\tau}^{\infty} (1 - T)\kappa(t)P(t)e^{-(1 - T)r(t - \tau)}dt \tag{7}$$

Let entrepreneurs expect physical marginal productivity of capital stock to be growing at the stationary rate g_κ:

$$\kappa(t) = \kappa(\tau)e^{g_\kappa(t - \tau)}$$

and price of output to be growing at the stationary rate g_p:

$$P(t) = P(\tau)e^{g_p(t - \tau)}$$

Insert these into (7), define

$$\rho \equiv (1 - T)r - (g_\kappa + g_p) \qquad\qquad (8)$$

and write the integral (7) as

$$k(\tau) = \int_\tau^\infty (1 - T)\kappa(\tau)P(\tau)e^{-\rho(t - \tau)}dt$$

Neither $(1 - T)$, $\kappa(\tau)$, nor $P(\tau)$ is a function of t; hence they may be taken outside the integral sign. Our g_κ, g_p, and r were all said to be stationary; hence the coefficient ρ of t is stationary, too. Assume $\rho > 0$. As a result find the integral to be

$$k = (1 - T)\kappa P/\rho \qquad\qquad (9)$$

Find present net worth of another physical unit of capital stock as its gross worth *minus* its price:

$$n \equiv k - P = [(1 - T)\kappa/\rho - 1]P$$

Desired capital stock is the size of stock for which the present net worth of another physical unit of capital stock equals zero, or

$$(1 - T)\kappa = \rho \tag{10}$$

Finally take equations (6) and (10) together and find desired capital stock

$$S = (1 - T)\beta X/\rho \tag{11}$$

In accordance with the definition (2) take the growth rate of desired capital stock (11) and write desired investment

$$I \equiv g_S S = (1 - T)\beta g_S X/\rho \tag{12}$$

If we think of (11) and (12) as being derived for an individual entrepreneur, then everything except X on their right-hand sides is common to all entrepreneurs. Factor out all common factors, sum over all entrepreneurs, then X becomes national physical output, and (11) and (12) become national desired capital stock and investment, respectively.

For all their attention to crowding-out, monetarists have shown little interest in deriving investment functions like (12). But let us take a closer look at (11) and (12) just the same. Both are in inverse proportion to ρ. What is ρ? In the definition (8) of ρ, let it be correctly foreseen that $g_\kappa = 0$--our steady-state growth and inflation model will indeed have the solution (27), and historically the physical marginal productivity κ has displayed no secular trend. In that case ρ collapses into the aftertax real rate of interest $(1 - T)r - g_p$.

In the special case of taxation but no inflation, $g_p = 0$, (10) will collapse into the familiar Keynesian case $\kappa = r$. In the special case of inflation but no taxation, $T = 0$, (10) will collapse into the familiar Fisherian case $\kappa = \rho = r - g_p$. But with both inflation and taxation present, nothing less than (10) will do.

Capital stock was assumed to be immortal, so we may ignore capital consumption allowances and define national income as the market value of physical output

$$Y \equiv PX \qquad\qquad (13)$$

Let all such national income be distributed to persons and be taxed once and at the rate T. Then tax revenue is

$$R = TY \qquad\qquad (14)$$

where $0 < T < 1$.

Define disposable income as national income _minus_ tax revenue:

$$y \equiv Y - R \qquad\qquad (15)$$

Let consumption be the fraction c of disposable real income:

$$C = cy/P \qquad\qquad (16)$$

where $0 < c < 1$.

Let government finance its deficit, if any, by increasing the money supply. The government budget constraint is then

$$GP - R = g_M M \qquad\qquad (17)$$

Let labor employed be the proportion λ of available labor force:

$$L = \lambda F \qquad\qquad (18)$$

where $0 < \lambda \leq 1$ is the "natural" rate of employment.

Let the demand for money be a function of money national income
and of the aftertax nominal rate of interest:

$$D = mY[(1 - T)r]^{\mu} \tag{19}$$

where $\mu < 0$ and $m > 0$.

Goods-market equilibrium requires the supply of goods to equal the
demand for them:

$$X = C + I + G \tag{20}$$

Money-market equilibrium requires the supply of money to equal
the demand for it:

$$M = D \tag{21}$$

We may now proceed to solving the system for the growth rates of
its variables and for the level of its aftertax real rate of interest.

III. SOLUTIONS

1. Growth-Rate Solutions

Consider our natural rate λ a stationary parameter, insert (18) into (3), take growth rates, and find

$$g_X = g_a + \alpha g_F + \beta g_S \qquad (22)$$

Insert (13), (14), (15), (16), (17) and the definitional part of (12) into (20), rearrange, and find the rate of growth of physical capital stock

$$g_S = [(1 - c)(1 - T) - g_M M/Y]X/S \qquad (23)$$

For the following reasons the square bracket of (23) is stationary. First, the growth rate g_M of the money supply was said to be stationary. According to (19) and (21) $M/Y = m[(1 - T)r]^\mu$. Second, all parameters c, m, μ, and T were said to be stationary. Third, the nominal rate of interest r was assumed to be stationary.

Consequently the growth rate of the growth rate (23), i.e., the rate of acceleration of physical capital stock is simply

$$g_{gS} = g_X - g_S$$

Insert (22) and write the rate of acceleration as

$$g_{gS} = \alpha(g_a/\alpha + g_F - g_S) \tag{24}$$

In (24) there are three possibilities: if $g_S > g_a/\alpha + g_F$ then $g_{gS} < 0$. If

$$g_S = g_a/\alpha + g_F \tag{25}$$

then $g_{gS} = 0$. Finally, if $g_S < g_a/\alpha + g_F$, then $g_{gS} > 0$. Consequently, if greater than (25) g_S is falling; if equal to (25) g_S is stationary; and if less than (25) g_S is rising. Furthermore, g_S cannot alternate around (25), for differential equations trace continuous time paths, and as soon as a g_S-path touched (25) it would have to stay there. Finally, g_S cannot converge to anything else than (25), for if it did, by letting enough time elapse we could make the left-hand side of (24) smaller than any arbitrarily assignable

positive constant ε, however small, without the same being possible for the right-hand side. We conclude that g_S must either equal $g_a/\alpha + g_F$ from the outset or, if it does not, converge to that value.

Once such convergence has been established we may easily find the corresponding values of other growth rates: insert (25) into (22), recall that $\alpha + \beta = 1$, and find the long-run growth rate of physical output

$$g_X = g_S \tag{26}$$

Take the growth rate of (6), insert (26), and find

$$g_\kappa = 0 \tag{27}$$

By taking growth rates of the entire system (1) through (21) the reader may convince himself that it is satisfied by (25), (26), and (27) as well as the following solutions

$$g_C = g_X \tag{28}$$

$$g_D = g_M \tag{29}$$

$$g_G = g_X \qquad\qquad (30)$$

$$g_I = g_X \qquad\qquad (31)$$

$$g_L = g_F \qquad\qquad (32)$$

$$g_M = g_Y \qquad\qquad (33)$$

$$g_R = g_Y \qquad\qquad (34)$$

$$g_r = 0 \qquad\qquad (35)$$

$$g_\rho = 0 \qquad\qquad (36)$$

$$g_{w/P} = g_a/\alpha \qquad\qquad (37)$$

$$g_Y = g_P + g_X \qquad\qquad (38)$$

$$g_y = g_Y \qquad\qquad (39)$$

Our growth was steady-state growth because no right-hand side of our solutions (25) through (39) was a function of time.

2. Properties of Growth-Rate Solutions: Inflation Control

Our growth-rate solutions deliver Friedman's (1968) conclusions.
First, no growth-rate solution for the nine real variables C, G, I, κ,
L, ρ, S, w/P, and X has the rate of growth g_M of the money supply in
it, directly or indirectly. Second, the growth-rate solutions for the
five nominal variables D, P, R, Y, and y have the rate of growth g_M
of the money supply in them, directly or indirectly. Specifically,
the rate of growth g_M of the money supply may be thought of as a
policy instrument used to control inflation: take the growth-rate
solutions (33) and (38) together, insert (25) and (26), and find

$$g_P = g_M - (g_a/\alpha + g_F) \tag{40}$$

or, in English, knowing the rate of technological progress g_a and the
rate of growth of the labor force g_F and knowing the elasticity α of
physical output with respect to labor, the monetary authorities may
control the rate of inflation g_P by controlling the rate of growth
g_M of the money supply.

3. The Aftertax Real Rate of Interest: Crowding-Out

Use (11) to express ρ as (1 - T)βX/S, insert (23), and find

$$\rho = \frac{\beta g_S}{1 - c - g_M(M/Y)/(1 - T)} \qquad (41)$$

A larger government purchase G may be financed either by a higher tax rate T or by a more rapidly growing money supply allowing, according to (17), a larger deficit at an unchanged tax rate. So either T or g_M is up. Whether T or g_M is up, the effect upon the aftertax real rate of interest (41) is the same: the last, negative, term of the denominator of (41) is up, either because its denominator $1 - T$ is down or because its numerator $g_M M/Y$ is up. Either way (41) is up.

The higher aftertax real rate of interest (41) will discourage investment (12). In other words there will be crowding out. We learn from (41) that crowding-out will result not only from a larger deficit but also from a higher tax rate.

IV. SUMMARY AND CONCLUSION

Monetarists wish to include the rate of inflation among their equilibrating variables. Any model admitting inflation as an

equilibrating variable will contain a derivative with respect to time,
hence will be dynamic, and will contain two additional equilibrating
variables—the nominal and the real rate of interest. Consequently,
a monetarist model must be a dynamic two-interest-rates model.
According to Friedman, monetary policy cannot peg the rate of unemploy-
ment for more than very limited periods. Consequently, a monetarist
model must dismiss and go beyond such limited periods and become a
long-run model.

A long-run, dynamic, two-interest-rates model is obviously incom-
patible with the short-run, static, one-interest-rate IS-LM framework
offered by Friedman (1970) himself as his "theoretical framework." As
Thygesen (1977) observed in his Nobel article, Friedman "is clearly
uncomfortable with it." He should be!

By contrast we found a neoclassical steady-state growth model
capable of delivering most of Friedman's conclusions. His growth-<u>rate</u>
conclusions were delivered impeccably. No growth-rate solution for a
real variable included the rate of growth of the money supply. All
growth-rate solutions for nominal variables included the rate of
growth of the money supply. His growth-<u>level</u> conclusions—that money
shouldn't matter for the level of real variables—did not hold for the
level of the aftertax real rate of interest: it was up if g_M was up.

But in fact monetarists agree. In our model money came into existence by financing a budget deficit, and monetarists agree that such deficits produce crowding-out--indeed monetarists gave us the crowding-out concept. And how can crowding-out occur except _via_ a higher aftertax real rate of interest?

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